Tutorial: An Introduction to Hierarchical Task Network (HTN) Planning

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- heuristics, esp. delete relaxation
- search strategies (A*, greedy, etc.)
- basic complexity theory (Chomsky hierarchy, automata, etc.)

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- learn the most important solving techniques

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- learn basic theoretical properties of HTN planning: hardness of the problem(s), expressivity
- learn the most important solving techniques
- obtain some ideas on how to design heuristics (why is it more complicated than in non-hierarchical planning?)

Part I: Theoretical Foundations

- Problem Definition(s)
- Computational Complexity of Plan Existence Problem
- Expressivity Analysis

Part II: Practice

- Solution Techniques
- Heuristics
- Excursion: Further Hierarchical Planning Formalisms

Overview Part I

Theoretical Foundations

- **Introduction**
- Problem Definition
- Computational Complexity of the Plan Existence Problem
  - General HTN Planning
  - HTN Planning with Task Insertion
  - Totally Ordered HTN Planning
  - Restricting Recursion (Acyclic, Regular, Tail-recursive)
- Expressivity Analysis

Classical Planning (Recap)

Environment:

- Fully observable
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- Deterministic
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Planning:

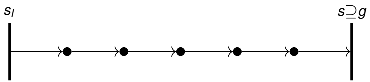
- Offline
- Usually ground and via progression search
- Solutions are action sequences

Classical Planning (Recap, cont'd)

Problem formalization, $\mathcal{P} = (V, s_I, A, g)$:

- Set of state variables V
- Initial state $s_I \in 2^V$
- Set of actions A, $a \in A$ has the form $(prec, add, del) \in (2^V)^3$
  An action $(prec, add, del)$ is executable in a state $s \in 2^V$ iff $prec \subseteq s$. Its application to s results into the state $(s \setminus del) \cup add$.
  Executability of task sequences defined analogously
- Goal description $g \subseteq V$

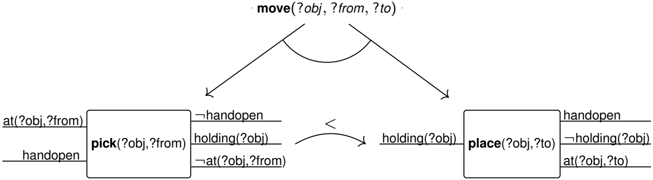
solution:



Informal Problem Definition

What is hierarchical planning, anyway?

Here: the model specifies a ***task hierarchy***: *compound* (or *complex*, *abstract*, *high-level*) tasks need to be decomposed into *primitive tasks*.



Problem given as a compound task (or a set of compound and/or primitive tasks).

Goal: Finding a (primitive) executable refinement.

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- Communicate plans on different levels of abstraction
- Incorporate task abstraction in plan explanations

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HTN Planning vs. Classical Planning

"HTN planners differ from classical planners in what they plan for and how they plan for it. In an HTN planner, the objective is not to achieve a set of goals but instead to perform some set of tasks."

(Ghallab, Nau, and Traverso; Automated Planning: Theory and Practice)

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Main differences to classical planning problems:

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Main differences to classical planning problems:

- It's not about generating some goal state! The goal is find a refinement of the initial task(s), not to satisfy some goal description
- There is no arbitrary task insertion: to alter task networks, we need to decompose compound tasks using their pre-defined methods

Problem Definition & Solution Criteria

$$\mathcal{P} = (V, P, \delta, C, M, s_I, c_I)$$

- V a set of state variables

Problem Definition & Solution Criteria

primitive
tasks

compound
tasks

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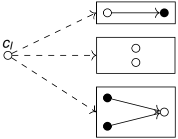

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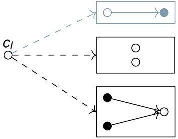

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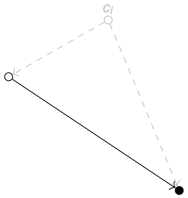

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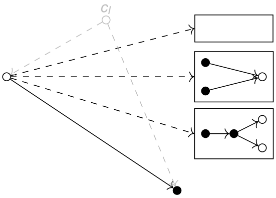

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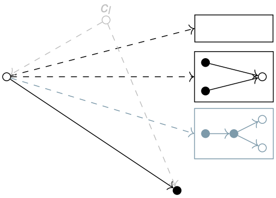

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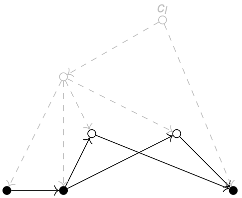

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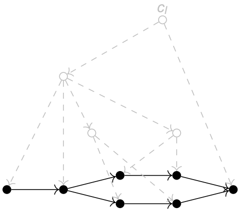

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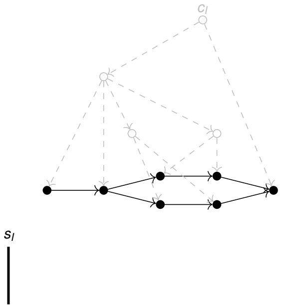

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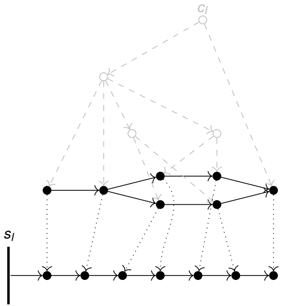

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- have an executable linearization.

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  - T, a possibly empty set of *tasks* or *task identifier symbols*
  - $\prec$, a partial order on the tasks
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  Primitive task names are mapped to their tuples by the task name mapping $\delta : P \rightarrow (2^V)^3$

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- Let $tn = (T, \prec, \alpha)$ be a task network, $t \in T$ a task identifier, and $\alpha(t) = c$ a compound task to be decomposed by $m = (c, tn_m)$. We assume $T \cap T_m = \emptyset$.
  Then, the application of m to tn results into the task network $tn' = ((T \setminus \{t\}) \cup T_m, \prec \cup \prec_m \cup \prec_X, \alpha \cup \alpha_m)|_{(T\setminus\{t\})\cup T_m}$ with:

  $$\begin{aligned}\prec_X := &\{(t', t'') \mid (t', t) \in \prec, t'' \in T_m\} \cup \\ &\{(t'', t') \mid (t, t') \in \prec, t'' \in T_m\}\end{aligned}$$

  where $(X_1, \ldots, x_n)|_Y$ restricts the sets X_i to elements in Y

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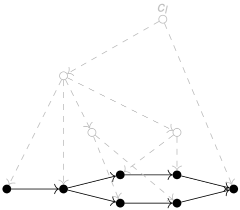

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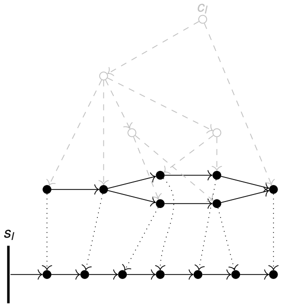

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Impact of Initial Task Network

Recap: $\mathcal{P} = (V, P, \delta, C, M, s_I, c_I)$ describes an HTN planning problem as described before.

Let $\mathcal{P}^\star = (V, P, \delta, C, M, s_I, \boldsymbol{tn_I})$ be an **HTN planning problem with initial task network $\boldsymbol{tn_I}$.**

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Then, a task network *tn* is a solution if and only if:

- There is a sequence of decomposition methods $\overline{m}$ **that transforms $\boldsymbol{tn_I}$ into $\boldsymbol{tn}$**,
- *tn* contains only primitive tasks, and
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Impact of Initial Task Network (Cont'd)

Theorem: Initial task networks can be compiled away.

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Proof:

Let $\mathcal{P}^\star = (V, P, \delta, C, M, s_I, \boldsymbol{tn_I})$ be an HTN planning problem with initial task network tn_I.

Then, there is an HTN planning problem $\mathcal{P}' = (V, P, \delta, \boldsymbol{C'}, \boldsymbol{M'}, s_I, \boldsymbol{c_I})$ with the same set of solutions:

Let $C' := C \,\dot{\cup}\, \{c_I\}$ and $M' := M \cup \{(c_I, tn_I)\}$.

Identical solution set is obvious.

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Let $\mathcal{P}^{\star} = (V, P, \delta, C, M, s_I, c_I, \boldsymbol{g})$ be an **HTN planning problem with goal description $\boldsymbol{g} \subseteq V$**.

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- **the task sequence $\overline{t}$ generates a goal state $\boldsymbol{s} \supseteq \boldsymbol{g}$.**

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Here, tn_I contains two tasks: c_I followed by a new primitive task p with no effects and g as precondition, $\delta(p) = (g, \emptyset, \emptyset)$.

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Then, the initial task network in $\mathcal{P}'$ can be compiled away as before.

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State Constraints in HTN Planning

State constraints have been introduced in the HTN formalization by Erol et al. (1994):

- (l, t), the literal l holds immediately before task t
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- (t, l, t'), the literal l holds in all states between t and t'

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Notably: Erol et al.'s formalization specifies a boolean constraint formula, in which *state*, *variable*, and *ordering constraints* can be specified with negations and disjunctions.

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Notably: Erol et al.'s formalization specifies a boolean constraint formula, in which *state*, *variable*, and *ordering constraints* can be specified with negations and disjunctions.

No compilation known yet.

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Given an HTN planning problem $\mathcal{P}$, does $\mathcal{P}$ possess a solution?

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Undecidability Proof

Theorem: HTN planning is undedicable.

Proof:

Reduction from the language intersection problem of two context-free grammars: given G and G', is there a word ω in both languages $L(G) \cap L(G')$?

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- Translate the production rules to decomposition methods. That way only words in $L(G)$ and $L(G')$ can be produced
- Any solution *tn* contains the word ω – encoded as action sequence – twice: once produced by G and once produced by G'. The action encodings ensure that no other task networks are executable

Undecidability Proof (Cont'd)

Theorem: HTN planning is undedicable.

Proof: (Cont'd, by example)

Let $G = (\overbrace{\Gamma = \{H, Q\}}^{\text{non-terminal symbols}}, \overbrace{\Sigma = \{a, b\}}^{\text{terminal symbols}}, \overbrace{R}^{\text{production rules}}, \overbrace{H}^{\text{start symbol}})$ and
$G' = (\Gamma' = \{D, F\}, \Sigma' = \{a, b\}, R', D)$.

Production rules R:	$H \mapsto aQb$	$Q \mapsto aQ \mid bQ \mid a \mid b$
Production rules R':	$D \mapsto aFD \mid ab$	$F \mapsto a \mid b$

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$$\mathcal{P} = (V, \overbrace{\{H, Q, D, F\}}^{C}, \overbrace{\{a, b, a', b'\}}^{P}, \delta, M, \overbrace{\{v_{turn:G}\}}^{\text{initial state}}, tn_I, \overbrace{\{v_{turn:G}\}}^{\text{goal description}})$$

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$$M = M(G) \cup M(G') \text{ (translated production rules of } G' \text{ and } G')$$

$$tn_I = (\underbrace{\{t, t'\}}_{T}, \underbrace{\emptyset}_{\prec}, \underbrace{\{(t, H), (t', D)\}}_{\alpha})$$

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Which properties make the plan existence problem easier?

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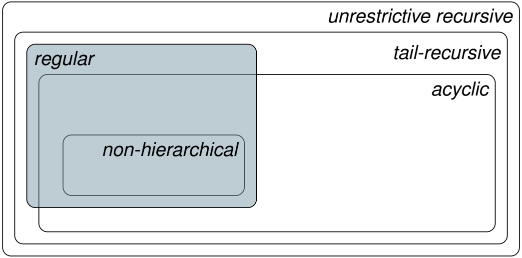

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In *HTN planning with task insertion*, *TIHTN planning*, tasks may be added arbitrarily to task networks (not just via decomposition):

Let $\mathcal{P}^\star = (V, P, \delta, C, M, s_I, c_I)$ be a **TIHTN planning problem**.

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Then, a task network *tn* is a solution if and only if:

- There is a sequence of decomposition methods $\overline{m}$ **and task insertions** that transforms c_I into *tn*,
- *tn* contains only primitive tasks, and
- the (still partially ordered) task network *tn* admits an executable linearization $\overline{t}$ of its tasks.

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Then, a task network *tn* is a solution if and only if:

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- $\boldsymbol{tn \supseteq tn'}$ **contains all tasks and orderings of** $\boldsymbol{tn'}$,
- *tn* contains only primitive tasks, and
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Motivation

Benefits of allowing task insertion:

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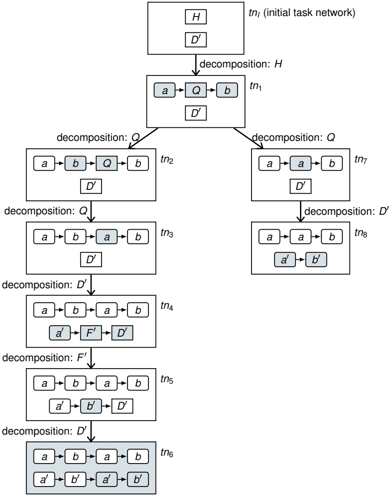
- Task insertion makes the modeling process easier: certain parts can be left to the planner
- Task insertion makes the problem computationally easier (can be exploited for heuristics)

Influence of Task Insertion



Recap: A task network is a solution if it contains the same word ω twice.

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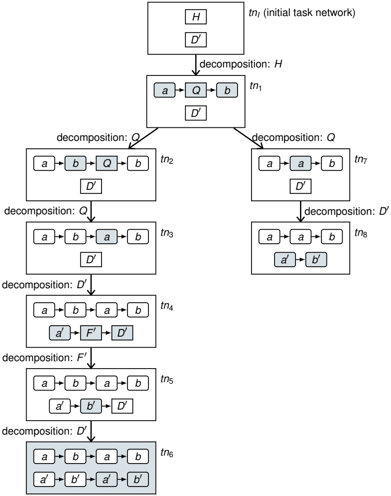

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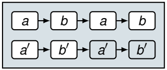
Task network tn_6 is a solution!



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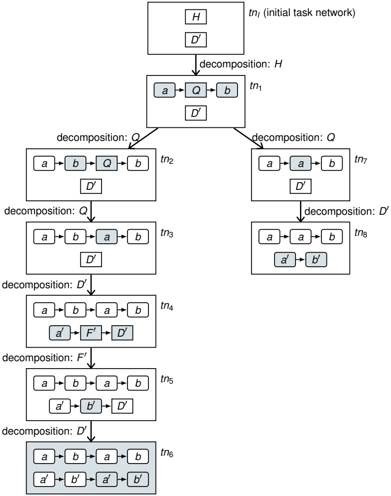

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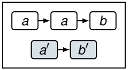
Task network tn_8 is no solution!



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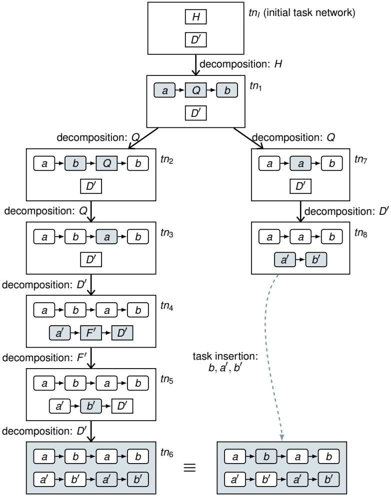

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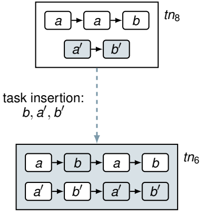

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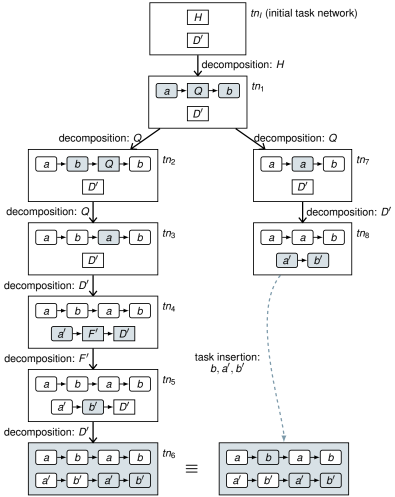

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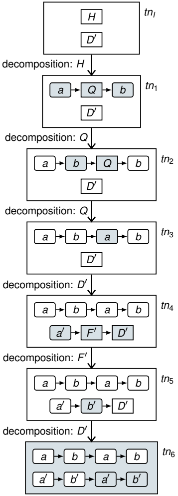
Observation:

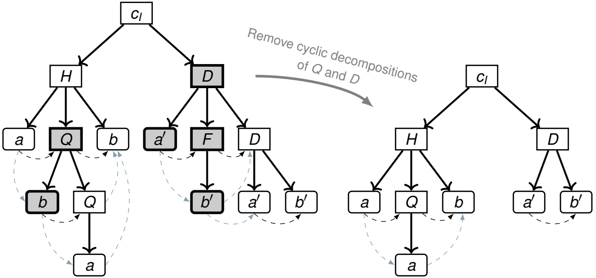
In TIHTN planning, recursion is not required.

Eliminating Recursion



Theorem: TIHTN planning is in **NEXPTIME**

Idea: Restrict to *acyclic* decompositions, fill the rest with task insertion, and verify.



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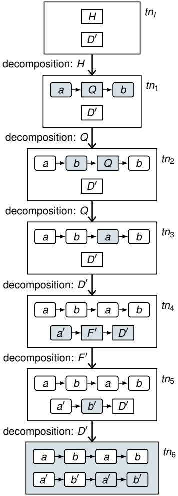

Theorem: TIHTN planning is in **NEXPTIME**

1. Step: Guess an acyclic decomposition:

The guessed decomposition tree describes at most $b^{|C|+1}$ decompositions.

(C = set of compound tasks)
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Verify in $O(b^{|C|+1})$ whether the tree describes a correct sequence of decompositions.

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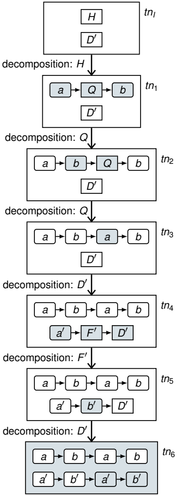

Theorem: TIHTN planning is in **NEXPTIME**

2. Step: Guess the actions and orderings to be inserted.

The (guessed) decomposition tree results into a task network with at most $\leq b^{|C|+1}$ tasks.

Between each two actions, at most $2^{|V|}$ actions need to be inserted to achieve the next precondition.

($|V|$ = number of state variables)

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An HTN planning problem $\mathcal{P} = (V, P, \delta, C, M, s_I, c_I)$ is called totally ordered if:

- All decomposition methods are totally ordered, i.e., for each $m \in M$, $m = (c, tn)$, tn is a totally ordered task network.

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- In case $\mathcal{P}$ uses an *initial task network* tn_I rather than an *initial task* c_I, then tn_I needs to be totally ordered as well.

Computational Complexity

Theorem: Totally ordered HTN planning is in **EXPTIME**

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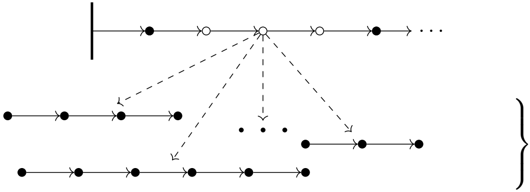
Intuition:

- Since plans are totally ordered, the only means of choosing the right refinement for a given compound task is to produce a suitable successor state



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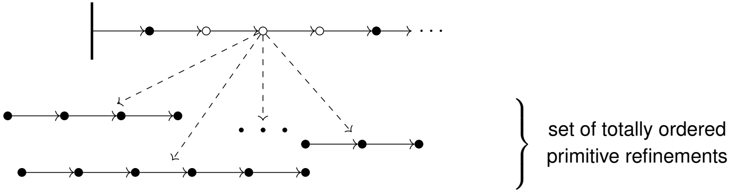

- There are only finitely many states that can be produced by the refinements of a given compound task

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An HTN planning problem is called called *acyclic* if no compound task can reach itself via decomposition.

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- Verify applicability of resulting linearization in $O(b^{|C|+1})$

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- A planning problem is called regular if all methods are regular.

Computational Complexity

Theorem: Regular problems are in **PSPACE**.

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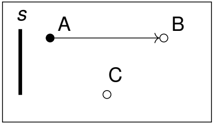
Proof:

- Rely on progression search

Excursion: HTN Progression Search



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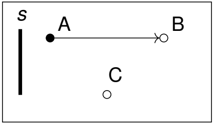

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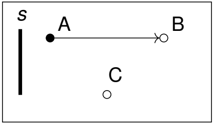
- Here, these are the tasks *A* and *C*.

Excursion: HTN Progression Search



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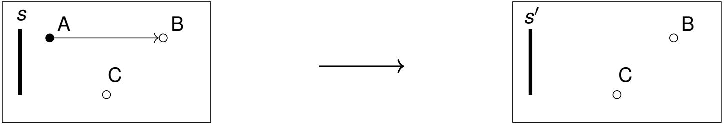

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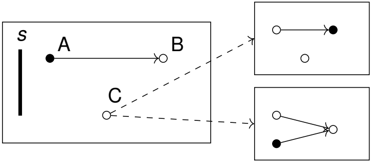

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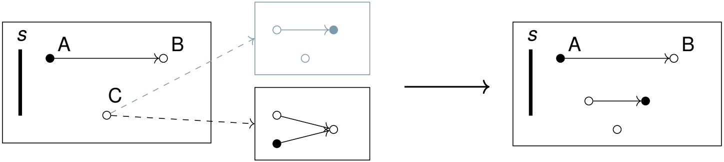

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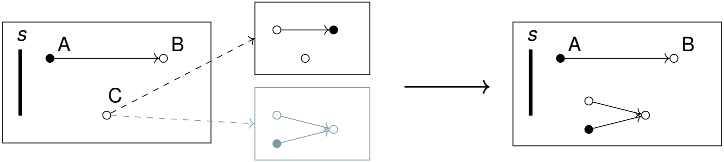

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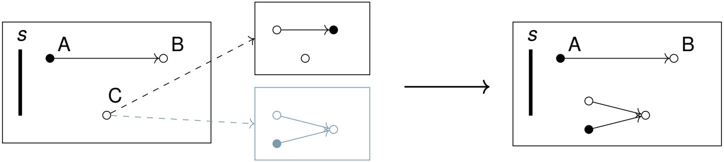

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- More details in Part II of this tutorial

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Theorem: Regular problems are in **PSPACE**.

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- That way, the size of any task network is bounded by the size of the largest task network in the model

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Every STRIPS problem $\mathcal{P}_{STRIPS}$ can be canonically expressed by a totally ordered regular HTN problem $\mathcal{P}$:

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- The initial task is X

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- Introduction
- Problem Definition
- Computational Complexity of the Plan Existence Problem
  - General HTN Planning
  - HTN Planning with Task Insertion
  - Totally Ordered HTN Planning
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Informal Problem Definition

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Formally, the restrictions on recursion are defined in terms of so-called *stratifications*.

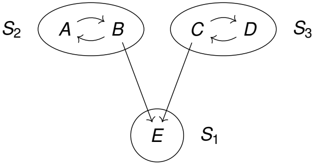
Stratifications

A stratification is defined as follows:

- A set $\leq \subseteq C \times C$ is called a *stratification* if it is a total preorder (i.e., reflexive, transitive, and *total*)

Stratifications: Example

Stratifications: (Non-)Examples



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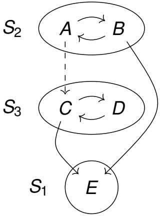
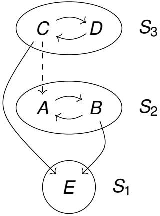


(b) Stratification $\leq_b$.



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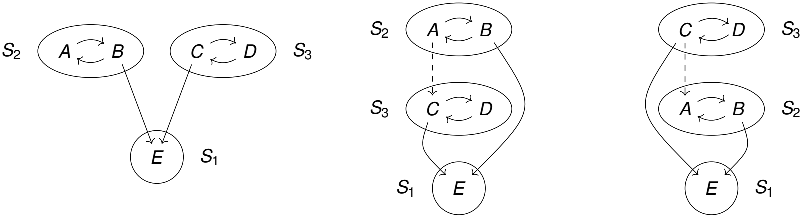

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- $\leq_a = \{(A,B),(B,A),(C,D),(D,C),(E,B),(E,C)\}$
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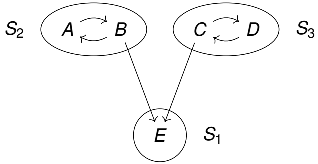

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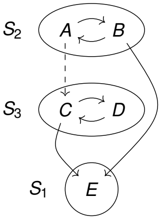

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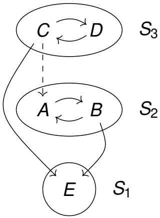

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- The *height of a stratification* is the number of its strata.

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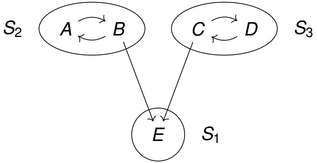

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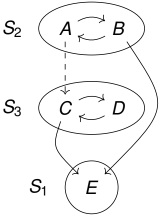

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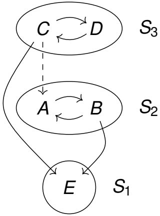

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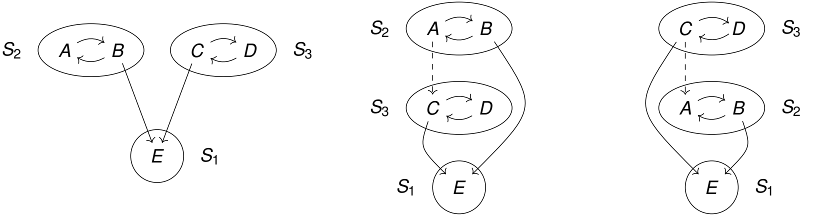

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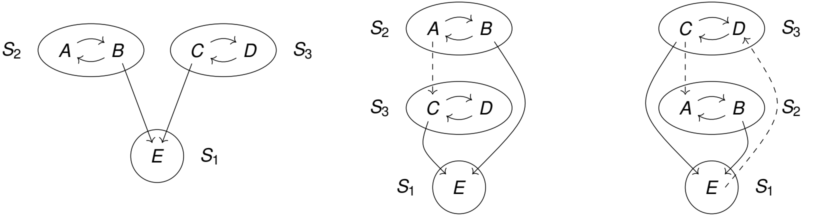

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- $\leq_b$ and $\leq_c$ have a height of 3.
- If we add, e.g., an edge from E to D in $\leq_c$, i.e., the tuple (D, E), then we only have *a single* stratification with height 1.

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An HTN problem $\mathcal{P}$ is called *tail-recursive* if there is a stratification $\leq$ on the compound tasks C of $\mathcal{P}$ with the following property:

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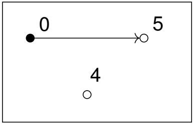

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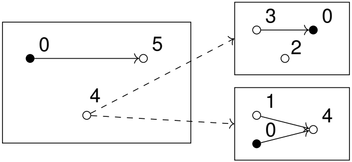

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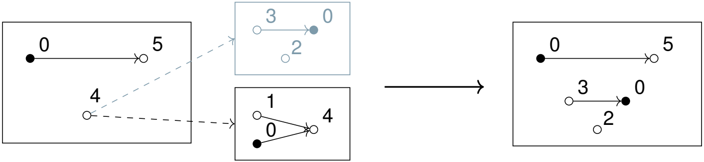

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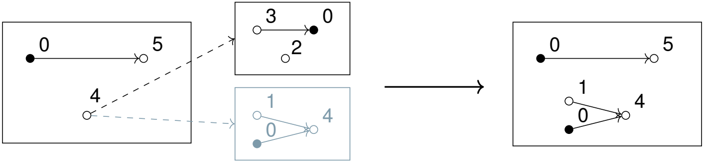

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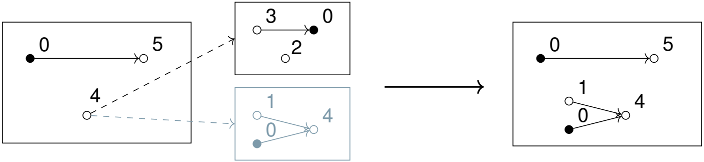

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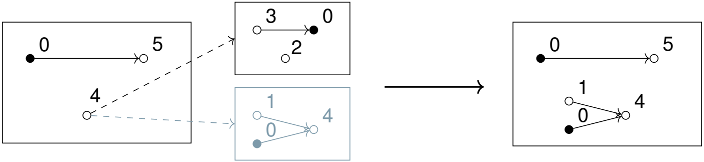

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- The decomposition of non-last tasks results into tasks of strictly lower stratum
- From this, we can calculate a *progression bound* – a maximal size of task network created under progression
- We get $k \cdot m^h$ as progression bound, where k is size of the initial task network, m is the size of the largest method, and h is the stratification height

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- When *task insertion* is allowed:
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Better answer:

- Formalism *A* can be compiled into formalism *B* (under several restrictions on compilation size and/or runtime)
- Gives an intuition on the relative expressivity
- Answer regarding STRIPS and HTN planning would be: (in general) impossible

Expressivity of Planning Formalisms

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- Provide a measure that allows more insights into the *structures* that can be represented

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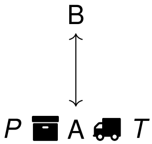

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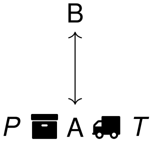

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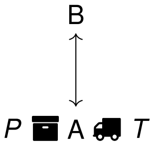

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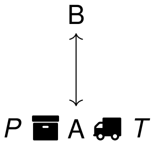
Expressivity of Planning Formalisms

In the next slides:

- Provide a measure that allows more insights into the *structures* that can be represented
- Consider the small STRIPS planning problem given at the right, P shall be delivered at B



- Model is a compact representation for a space of states
- Actions define state transitions
- Initial state and goal definition specifies a set of (transition) sequences we are interested in

  $\{\langle pickup(T,P,A), move(T,A,B), drop(T,P,B)\rangle,$

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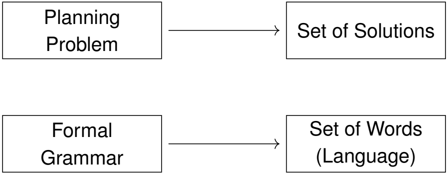
$\rightarrow$ The planning problem is a compact representation for a (possibly infinite) set of sequences

Comparison to Formal Languages



- Actions of a problem form the (terminal) symbols of a language
- Solution criteria define valid words
- Set of solutions forms the *language* of the problem

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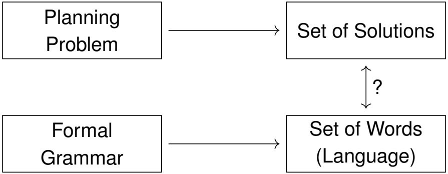

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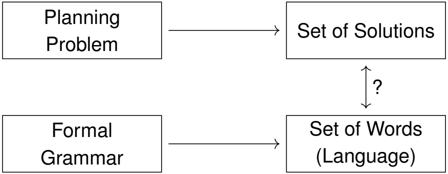

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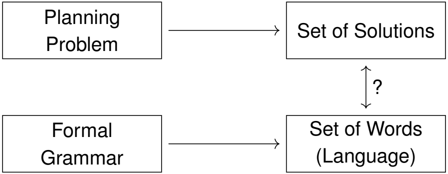
→ Which languages can be expressed using a certain formalism?

Comparison to Formal Languages



- Actions of a problem form the (terminal) symbols of a language
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$\rightarrow$ Which languages can be expressed using a certain formalism?

Question (given before):

- What can be *expressed* with the planning formalism at hand?

Answer:

- STRIPS can represent (a subset of the) regular languages

Expressivity via Comparison to Formal Languages

- Chomsky hierarchy as reference framework

Expressivity via Comparison to Formal Languages

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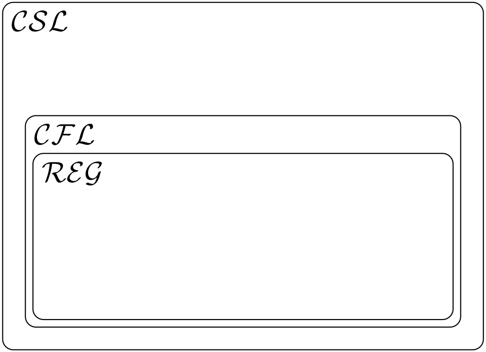

Language of STRIPS with Conditional Effects

Theorem: STRIPS with conditional effects (SCE) is equivalent to the regular languages

Language of STRIPS with Conditional Effects

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Proof: *(to show)*

- For every SCE planning problem, there is an equivalent regular language
- For every regular language, there is a SCE problem generating it

Language of STRIPS with Conditional Effects

- Let $P = (V, A, s_0, g)$ be a planning problem
- We define a Deterministic Finite Automaton (Σ, S, d, i, F) with
  - Σ is its input alphabet
  - S its set of states
  - $d : S \times \Sigma \to S$ its state-transition function
  - i its initial state
  - F its set of final states

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- We define
  - $\Sigma = A$
  - $S = 2^V$
  - i contains exactly the literals that hold in s_0
  - Every state including the literals in g is included in F

$$d(s, a) = \begin{cases} s', & \textit{iff } (\tau(a, s) \wedge \gamma(a, s) = s') \\ \textit{undefined}, & \textit{else} \end{cases}$$

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$$\forall a \in A : prec(a)$$

$$add(a)$$

$$del(a)$$

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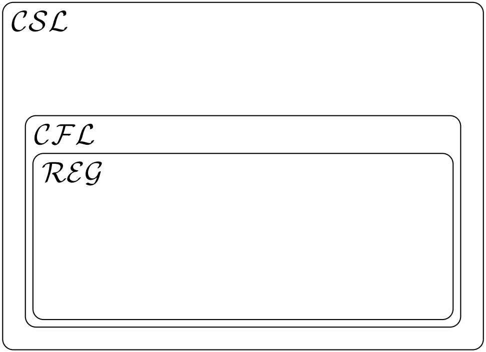
$$del(a) = \{(\emptyset \rightarrow V)\}$$

Expressivity via Comparison to Formal Languages



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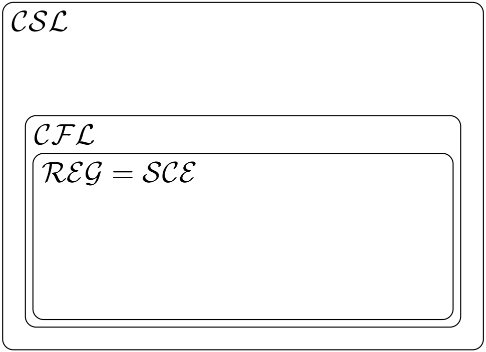
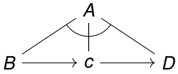


Totally Ordered HTN Planning Problems

- Decomposition in totally ordered HTN planning problems is very similar to rule application in context-free grammars



$$A \mapsto BcD$$

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$$\begin{tikzcd} & A \arrow[dl, dash] \arrow[d, dash] \arrow[dr, dash] & \\ B \arrow[r] & c \arrow[r] & D \end{tikzcd} \qquad A \mapsto BcD$$

- The encoding of (totally ordered) HTN decomposition as (context-free) grammar rules and vice versa is straightforward
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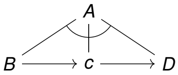

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$\rightarrow \mathcal{HTN}{-}\mathcal{ORD} = \mathcal{CFL}$

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$\mathcal{CSL}$

$\mathcal{CFL}$

$\mathcal{REG} = \mathcal{SCE}$

Expressivity via Comparison to Formal Languages

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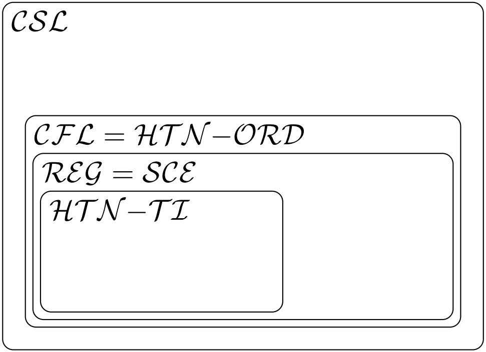

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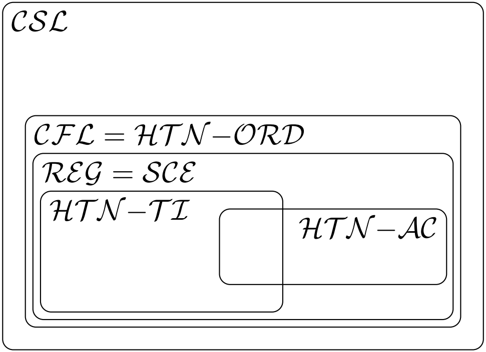

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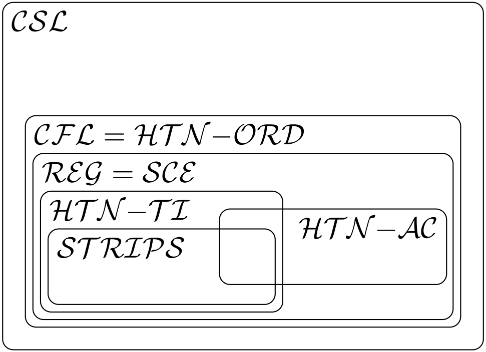

Noop HTN Planning Problems

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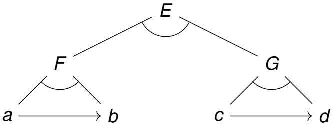
- First class we look at:
  $\mathcal{HTN}{-}\mathcal{NOOP}$ – actions have no preconditions and effects
- Can a partially ordered method be transfered to a set of totally ordered methods?

Noop HTN Planning Problems

HTN

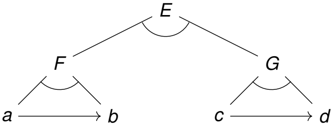


Grammar

$$E \mapsto FG \quad F \mapsto ab$$
$$E \mapsto GF \quad G \mapsto cd$$

Noop HTN Planning Problems

HTN



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Word 1 *cdab*

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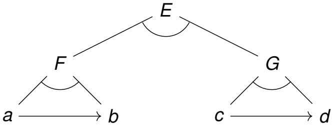

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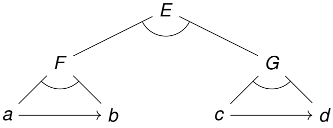

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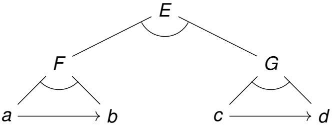

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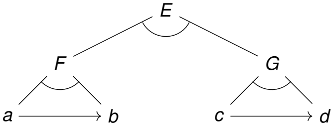

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Word 1 *cdab* ✓

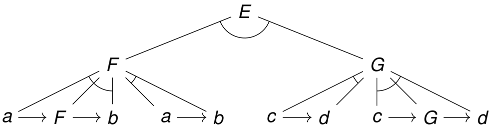
$ab||cd$

Word 2 *acbd* ✗

$\{abcd\} \cup \{cdab\}$

Noop HTN Planning Problems

- The HTN depicted below generates the language $a^n b^n || c^m d^m$
- Using the *Pumping Lemma* for context-free languages, it can be shown that this language is not context-free



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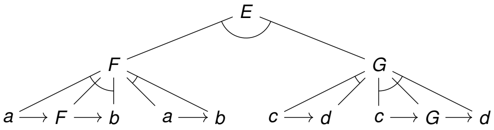
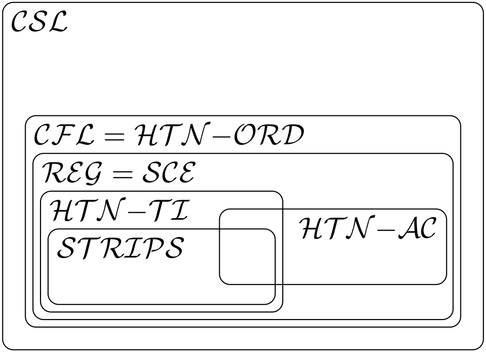
$\rightarrow$ $\mathcal{CFL} \subsetneq \mathcal{HTN}\text{–}\mathcal{NOOP}$



Expressivity via Comparison to Formal Languages



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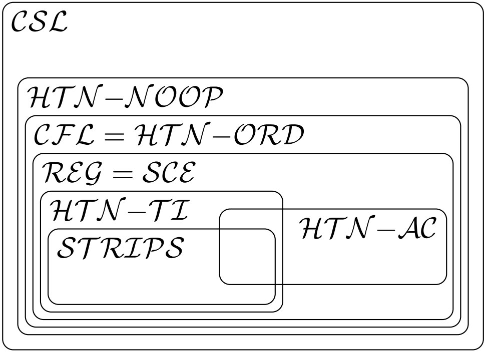

Full HTN Planning Problems

- For every HTN there is a linear space-bounded Turing machine that decides its word problem

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$\rightarrow$ $\mathcal{HTN} \subseteq \mathcal{CSL}$

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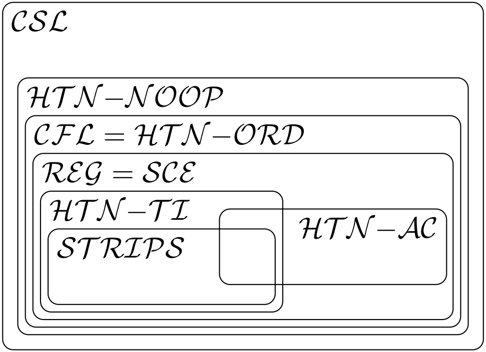

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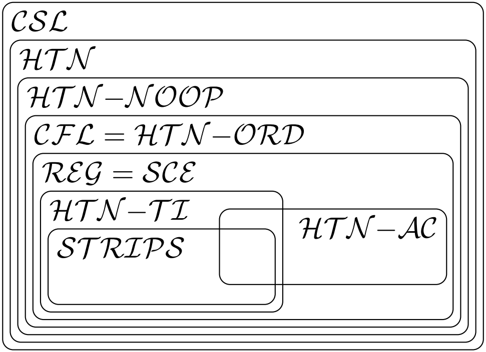

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- Chomsky hierarchy as reference framework
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  - Plan Recognition and Prefix Problem
- Representation blow-up is not considered
  - Theoretical approach to assess expressivity
  - Measures expressivity, not computational complexity

Thank You for Your Attention!

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Are there questions?